The Information Content of Option-implied Tail Risk on the Future Returns of the Underlying Asset

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ABSTRACT

We compile option-implied tail loss and gain measures based upon a deep out-of-the-money option pricing formula derived by applying 'extreme value theory', and then use these measures to investigate the information content of option-implied tail risk on the future returns of the underlying assets. Our empirical analysis based upon the S&P 500 index and the VIX shows that both tail measures implied by S&P 500 and VIX options can predict future changes in the corresponding underlying assets, with the tail loss (gain) measure being more informative than the tail gain (loss) measure for the S&P 500 index (VIX), and the relationships being particularly strong during periods of economic recession. Both tail measures compiled from S&P 500 and VIX options are also found to be informative on the future returns of the S&P 500 index, with the former providing the stronger contribution. Further evidence shows that these predictive relationships are driven by the tail-risk premium and quite short-lived.

Keywords: Tail measures; S&P 500; VIX; Options; Extreme value theory.

JEL Classification: G13; G14.

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1. INTRODUCTION

Trading in the derivatives markets is widely recognized as containing forward-looking information, with a number of studies having empirically investigated whether the information gleaned from the options markets reflects the future price dynamics of the underlying assets.[1] Within the extant related literature, aside from the focus on implied volatility,[2] some option-implied information proxies, such as implied correlations and market variance risk premium, have also been shown to be useful predictors of market returns;[3] however, all of these measures represent the general expectations on the price distribution of the underlying assets.

In one particular study, Pan and Poteshman (2006) demonstrated that the trading of options with higher leverage tended to be more informative with regard to the future dynamics of the underlying asset. This provided signals that the information relating to the tail properties of the price distribution of the underlying asset could be useful in determining its future dynamics.

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1 Prior studies on the prediction of returns with option-implied information include Whaley (2000), Giot (2005), Banerjee, Doran and Peterson (2007), Bakshi, Panayotov and Skoulakis (2011), Feunou, Fontaine, Taamouti and Tedongap (2014), Anersen, Fasari, and Todorov (2015) and Bollerslev, Todorov and Xu (2015). Christoffersen, Jacob and Chang (2013) also provided a comprehensive survey on option-implied information in forecasting.

2 The CBOE VIX index is invariably used as the proxy for implied volatility; this is compiled from the market prices of S&P 500 index options as the means of approximating the expected aggregate volatility of the S&P 500 index during the subsequent 30 calendar-day period. Following Whaley (2000), in which the VIX was found to be an effective ‘fear gauge’, Giot (2005) found a strong negative correlation between the contemporaneous changes, along with a positive relationship between the current levels of the implied volatility indices and future market index returns. Similar findings were also reported by Guo and Whitelaw (2006) and Banerjee et al. (2007).

3 Detailed explanations of these market return predictors are provided in Bollerslev, Tauchen and Zhou (2009) and Buss and Vilkov (2012).

Against this backdrop, we set out in this study to explore the way in which option-implied tail information can be extracted, and whether the information is of use in predicting the price dynamics of the underlying asset. Although the majority of the prior studies have tended to adopt approaches based upon the central distribution of the underlying asset as the means of extracting option-implied information, some of the more recent studies have focused their attention on the information implied in the tail of the price distribution.

Du and Kapadia (2012) constructed a jump index using the difference between the CBOE VIX and the model-free volatility proposed by Bakshi, Kapadia and Madan (2003), which was found to be capable of predicting the future returns of the S&P 500 index. Andersen, Fuasir and Todorov (2015) provided evidence to show that the tail factor is a critical element in the forecasting of the monthly market returns, as opposed to future volatility or jump risks. Bollerslev et al. (2015) also demonstrated that the left-tail jump risk premium was capable of predicting monthly market returns.

According to 'extreme-value theory' (EVT), the option pricing model is derived from certain distribution approximations which remain valid regardless of the true distribution of the underlying asset price. Therefore, following the theoretical framework of Hamidieh (2011),[4] we investigate whether the tail risk information

[4] Hamidieh (2011) derived a new option pricing formula based upon extreme-value theory as the means of estimating the tail shape parameter of the risk neutral density.

extracted from the out-of-the-money (OTM) options in the S&P 500 index and the VIX can be of use in predicting the dynamics of the corresponding underlying asset.

Given that numerous empirical observations have revealed a negative relationship between the VIX and the S&P 500 index, we examine whether the tail risk information implied in VIX options can also provide a predictive function with regard to the dynamics of the S&P 500 index, and if so, whether its information content overlaps with that of the information implied in the S&P 500 options. In specific terms, using the EVT-based deep-OTM option-pricing model proposed by Hamidieh (2011), we extend the method of Vilkov and Xiao (2013) to calculate the tail loss measure (TLM) and tail gain measure (TGM) from the option prices, and then go on to compile the respective measures from the S&P 500 and VIX options for subsequent empirical analysis.

If the tail measures do indeed provide information content on the future returns or levels of the underlying asset, then we would expect to find a positive predictive relationship for the tail gain measure, as compared to a negative relationship for the tail loss measure. Alternatively, if the tail measures represent the levels of tail risk, and investors require compensatory premiums for taking the risk, then we would expect to find a positive predictive relationship for both the tail gain and loss measures. Our main empirical results reveal the following.

Firstly, both the tail loss and gain measures compiled from S&P 500 index options are found to have positive associations with the future returns of the S&P 500 index, with this relationship being found to be stronger for the tail loss measure, and the information content of these tail measures differing from that of the VIX, although the tail measures are found to be highly correlated with the VIX. Secondly, both the tail loss and gain measures compiled from VIX options are found to positively predict the VIX level, with the effect being stronger for the tail gain measure.

Thirdly, the S&P 500 tail loss measure and the VIX tail gain measure are both found to provide significant information on the future returns of the S&P 500 index, with the effect being stronger for the S&P 500 tail loss measure. Fourthly, almost all of the predictive relationships are found to be particularly robust during periods of economic recession. Finally, we show that all of the predictive relationships are driven by the tail-risk premiums and that they are generally quite short-lived.

The primary objective of this study is to investigate whether the option-implied tail measures provide information with predictive ability on the future returns of the underlying asset. Among those studies investigating the effects of option-implied information content on the future dynamics of the underlying asset, greater focus is currently being placed on the option-implied tail information.[5] We follow the

[5] See for example, Du and Kapadia (2012), Andersen, Fusari and Todorov (2015), Bollerslev et al. (2015) and Park (2015).

methodology of Hamidieh (2011) and Vikov and Xiao (2013) to select the Black-Scholes-Merton model (the most popular and concise framework) and then use EVT to obtain a deep-OTM option-pricing equation. This provides us with a new channel for the estimation of the tail shape and scale parameters through which a conditional tail loss or gain can be identified (Vilkov and Xiao, 2013).

We adopt the Black-Scholes-Merton model essentially because we wish to avoid potential estimation errors due to too many parameters, and in contrast to the studies undertaken by Andersen et al. (2015) and Bollerslev et al. (2015), rather than monthly data frequency, the frequency used in the present study is set at the daily level. We also contribute to the extant literature by extending the research to cover the implied tail measures from VIX options, investigating the joint information content of S&P 500 and VIX option implied tail measures on the future dynamics of the S&P 500 index, and considering the impacts of the business cycle on the tail risk.

The remainder of this paper is organized as follows. Section 2 provides discussions on extreme value theory and the deep-OTM option-pricing formula, followed in Section 3 by a review of the literature and the development of our testable hypotheses. A data description and the control variable definitions are presented in Section 4, with Section 5 presenting the main empirical analysis, followed by further discussion in Section 6. Finally, the conclusions drawn from this study are presented in Section 7.

2. THEORETICAL FOUNDATION

2.1 Extreme Value Theory

Unlike the conventional option pricing models in which the entire distribution of the asset returns is specified, our model focuses on the tail distribution in order to obtain information on expected substantial changes in the asset returns. Let us suppose that random variables $\{X_i\}_{i=1}^{n}$ are independent and identically distributed with a distribution function, *F*, which belongs to the 'maximum domain of attraction' (MDA) of an extreme value distribution, *H*. This implies the existence of sequences $\{c_n\}$ and $\{d_n\}$, such that the normalized maximum order statistic, $\frac{max\, X_i - c_n}{d_n}$, converges to the non-degenerate distribution function, *H*, as $n \to \infty$. The only possible non-degenerate limiting distributions of *H* are in the 'generalized extreme value' (GEV) distribution family; that is, 'generalized Pareto distribution' (GPD).

According to the Pickands-Balkema-De Haan theorem (also referred to as the second theorem of extreme value theory) and the fact that *F* belongs to the MDA of *H*, the GPD would approximate the distribution of the random variable, *X*, above a sufficiently extreme threshold, μ, $X - \mu \mid X > \mu$; thus, a generalized Pareto distribution can be expressed as:

$$G_{\beta,\xi}(x-\mu) = \begin{cases} 1 - \left(1 + \xi \dfrac{x-\mu}{\beta}\right)^{-\frac{1}{\xi}}, & \xi \neq 0 \\ 1 - exp\left(-\dfrac{x-\mu}{\beta}\right), & \xi = 0, \end{cases}$$

where β (ξ) is the scale (tail) shape parameter for the right tail.

Similarly, we can approximate the distribution of the random variable, X, which is below a sufficiently extreme threshold h, $h - X \,|\, X < h$, as:

$$G_{\beta^*,\xi^*}(h-x) = \begin{cases} 1 - \left(1 + \xi^* \dfrac{h-x}{\beta^*}\right)^{-\frac{1}{\xi^*}}, & \xi^* \neq 0 \\ 1 - exp\left(-\dfrac{h-x}{\beta^*}\right), & \xi^* = 0 \end{cases}$$

where β^* (ξ^*) is the scale (tail) shape parameter for the left tail.

These approximations are valid regardless of the true distribution of the random variable (for example, the underlying asset price).[6] Therefore, if we assume that the conditional expectations are $\xi < 1$ or $\xi^* < 1$, then the respective mean excess right- and left-tail value of the underlying asset price, X, can be shown as:

$$E(X - \mu | X > \mu) = \frac{\beta}{1-\xi} \tag{1a}$$

and

$$E(h - X | X < h) = \frac{\beta^*}{1-\xi^*}. \tag{1b}$$

This implies that the extreme (right-tail) gain or (left-tail) loss are independent of the choice of critical points, μ and h, under the framework.

2.2 Tail Measures for the S&P 500 Index and VIX

Applying the above definition of conditional expectations to the S&P 500 index, the

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[6] Resnick (1987) demonstrated that almost all continuous distributions within the financial literature belonged to the maximum domain of attraction (MDA) of an extreme value distribution G. McNeil, Frey and Embrechts (2015) noted that if X belonged to the MDA of G, then $X - u \,|\, X > u$ (where u is sufficiently extreme) can be approximated by the GPD.

tail loss and gain measures are defined as the expected excess value conditional on a sufficiently extreme threshold relative to the current level of the S&P 500 index. However, for the VIX, these two measures are defined as the expected excess value conditional on a sufficiently extreme threshold, *per se*, since volatility has mean reversion, and thus, the absolute level is more economically meaningful than the value relative to the current level.

The tail loss measure (*TLM*) and tail gain (*TGM*) measure for the S&P 500 index and VIX are respectively denoted as TLM_{SPX}, TGM_{SPX}, TLM_{VIX} and TGM_{VIX}; thus, the tail loss and gain measures for the S&P 500 index are expressed as:

$$TLM_{SPX,t} = \frac{E(h - S_T | h > S_T)}{S_t} = \frac{\beta^*_{SPX}}{(1-\xi^*_{SPX})S_t}, \tag{2a}$$

and

$$TGM_{SPX,t} = \frac{E(S_T - \mu | S_T > \mu)}{S_t} = \frac{\beta_{SPX}}{(1-\xi_{SPX})S_t}, \tag{2b}$$

where S_t is the level of the S&P 500 index at time *t*.

The tail loss and gain measures for the VIX are expressed as:

$$TLM_{VIX,t} = E(h - VIX_T | h > VIX_T) = \frac{\beta^*_{VIX}}{1-\xi^*_{VIX}}, \tag{3a}$$

and

$$TGM_{VIX,t} = E(VIX_T - \mu | \mu - VIX_T) = \frac{\beta_{VIX}}{1-\xi_{VIX}}, \tag{3b}$$

where VIX_t is the level of the VIX at time *t*; and ξ^*_{SPX}, β^*_{SPX}, ξ^*_{VIX} and β^*_{VIX} (ξ_{SPX}, β_{SPX}, ξ_{VIX} and β_{VIX},) are the left- (right-) tailed parameters corresponding to the underlying asset (S&P 500 or VIX) at the specified time *t*.

2.3 Option Pricing Formulae and Parameter Estimation

As shown in Hamidieh (2011), the out-of-the-money (OTM) option pricing formulae for calls and puts are derived as:

$$C(K_i) = C(K) * \left(\frac{\xi}{\beta}(K_i - K) + 1\right)^{1-1/\xi} \tag{4}$$

and

$$P(K_i^*) = P(K^*) * \left(\frac{\xi^*}{\beta^*}(K^* - K_i^*) + 1\right)^{1-1/\xi^*}, \tag{5}$$

where $C(K)$ ($P(K)$) is the call (put) price at exercise price K; ξ (ξ^*) and β (β^*) are the respective tail and scale parameters for the right (left) side of the distribution; K (K^*) is the strike price which is sufficiently small (large) to qualify as the suitable threshold in the right (left) side; and K_i (K_i^*) is greater (less) than K (K^*) and increasing (decreasing) with index i.

Following the method of Hamidieh (2011), we calibrate the tail shape and scale parameters from the OTM option prices. For each trading day and maturity period, we use the following least squared error minimization to obtain the right (left) tail shape, $\hat{\xi}$ ($\hat{\xi}^*$), and scale parameters, $\hat{\beta}$ ($\hat{\beta}^*$):

$$\{\hat{\xi}, \hat{\beta}\} = argmin_{\xi,\beta} \sum_{i=1}^{n} \left(C_{MKT}(K_i) - C(K_i)\right)^2, \tag{6}$$

and

$$\{\hat{\xi}^*, \hat{\beta}^*\} = argmin_{\xi^*,\beta^*} \sum_{i=1}^{n^*} \left(P_{MKT}(K_i^*) - P(K_i^*)\right)^2, \tag{7}$$

where $P_{MKT}(K_i^*)$ (C_{MKT} (K_i)) is the observed or market option price of a put (call) with strike K_i; and $P(K_i^*)$ ($C(K_i)$) is the corresponding theoretical price of a put (call), according to Equations (4) and (5).

Let us suppose that there are n (n^*) different strike price levels in the OTM call (put) sample; that is, K_i (K_i^*) with $i = 1, 2, \cdots, n\,(n^*)$. For the S&P 500 index, K and K^* are chosen as the 30 per cent quantile of OTM strike prices of call and put options, respectively; thus, the *TLM* and *TGM* can be obtained as:[7]

$$TLM_{SPX,t} = \frac{\hat{\beta}^*_{SPX}}{(1-\hat{\xi}^*_{SPX})S_t} \tag{8}$$

and

$$TGM_{SPX,t} = \frac{\hat{\beta}_{SPX}}{(1-\hat{\xi}_{SPX})S_t}, \tag{9}$$

where $\{\hat{\xi}^*_{SPX}, \hat{\beta}^*_{SPX}\}$ and $\{\hat{\xi}_{SPX}, \hat{\beta}_{SPX}\}$ are obtained by the respective S&P 500 index put and call prices. Considering the liquidity in VIX options, we select K and K^* as the 50 per cent quantile of the OTM strike prices of call and put options, respectively.

Similarly, the *TLM* and *TGM* implied by the VIX options can also be obtained as:

$$TLM_{VIX,t} = \frac{\hat{\beta}^*_{VIX}}{(1-\hat{\xi}^*_{VIX})} \tag{10}$$

and

$$TGM_{VIX,t} = \frac{\hat{\beta}_{VIX}}{(1-\hat{\xi}_{VIX})}, \tag{11}$$

where $\{\hat{\xi}^*_{VIX}, \hat{\beta}^*_{VIX}\}$ and $\{\hat{\xi}_{VIX}, \hat{\beta}_{VIX}\}$ are obtained based upon the respective VIX

[7] Separate tail measures are obtained for each maturity period on each day; the tail measure of the maturity period with the greatest volume, which is usually the nearest maturity, is then selected as our tail measure.

put and call prices.

3. LITERATURE REVIEW AND HYPOTHESIS DEVELOPMENT

The effectiveness of option-implied information in the forecasting of the price dynamics of the underlying asset, has been investigated in numerous related studies, with the use of implied volatility for the forecasting of the return volatility of the underlying asset having been shown to be the most successful application.[8] Several related studies have provided explorations of the association between option-implied information and the future returns of the underlying asset,[9] whilst other studies have relied upon the use of option prices across different moneyness levels and maturity periods to generate the prediction of the risk-neutral density of the underlying asset price.[10]

A comprehensive survey of the various financial forecasting studies based upon option-implied information was recently provided by Christensen, Jacobs and Chang (2013), who showed that option-implied information was generally found to be playing a useful and important role in the determination of the future price dynamics of the underlying asset. Considerable attention has also been focused on the impacts of extreme events over recent years following a series of financial crises, such as the

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8 See, for example, Christensen and Prabhala (1998), Blair, Poon and Taylor (2001) and Busch, Christensen and Nielsen (2011).

9 Examples include Whelay (2000), Giot (2005), Banajee, Doran and Peterson (2007), Bakshi, Panayotov and Skoulakis (2011), Feunou, Fontaine, Taamouti and Tedongap (2014), Anersen, Fasari, and Todorov (2015) and Bollerslev, Todorov and Xu (2015).

10 See Breeden and Litzenberger (1978), Jackwerth (1999) and Bliss and Panigirtzoglou (2002).

Russian default in 1998, the bankruptcy of Lehman Brother in 2008 and the European sovereign debt crisis in 2010; however, it is extremely difficult to measure tail risk based upon historical observations due to the rarity of extreme (or tail) events. Several recent studies have, nevertheless, attempted to use option prices to investigate the predictability of the tail risk.

Vilkov and Xiao (2013) examined S&P 500 index options based upon the Black-Scholes-Merton model and EVT using the Hamidieh (2011) deep-OTM option-pricing formula to calibrate the tail shape parameter and obtain a tail loss measure (that is, the conditional expected shortfall). Bollerslev and Todorov (2011) define a new fear index as the difference between the extreme left variance risk premium and the extreme right one implied in the prices of deep-OTM S&P 500 index options and the high-frequency prices of S&P 500 futures. Du and Kapadia (2012) subsequently constructed a model-free jump and tail index using the prices of the S&P 500 index options over a 30-day period.

Kelly and Jiang (2014) used cross-sectional stock prices (rather than option prices) to construct a new tail index and examine its relationship with future monthly stock returns. Bollerslev et al. (2015) subsequently found that the tail risk, defined as the jump risk premium, was capable of predicting aggregate monthly market returns, whilst Park (2015) adopted the methodology of the Chicago Board Options Exchange

(CBOE) with VIX option prices to obtain the model-free volatility-of-volatility (VVIX) as a proxy for tail risk. Finally, a number of studies have also considered the impact of tail risk in consumption-based asset pricing models,[11] and indeed, this does appear to be an effective way of gauging the overall tail risk from the options market, since it is likely that the risk is priced into the financial assets.

In this study, we focus on S&P 500 index OTM options to construct the tail loss measure (TLM_{SPX}) and tail gain measure (TGM_{SPX}) of the S&P 500 index. Given that option-implied information is informative, in terms of the future price dynamics of the underlying asset, if options investors anticipate a greater likelihood of large negative (positive) returns, then the price of the underlying asset will consequently move downwards (upwards); therefore, TLM_{SPX} (TGM_{SPX}) is expected to be negatively (positively) correlated with the future returns of the S&P 500 index. This viewpoint is generally referred to as the 'information hypothesis'.

However, if the tail risk is already priced into the financial assets, we would expect to find that the higher the tail risk, the higher the future returns; hence, we should anticipate a positive relationship, for both tails, between the level of the tail risk measure and the future changes in the underlying asset.[12] This viewpoint is referred to as the 'risk-premium hypothesis'. Given that, in terms of risk, investors

[11] Examples include Rietz (1988), Barro (2006), Gabaix (2008, 2012) and Wachter (2013).
[12] In essence, the S&P 500 index is not actually a tradable asset; however, the exchange traded funds (ETF) of the index have made it equivalent to a tradable asset.

are more concerned by negative than positive price changes, we expect to find the risk-premium hypothesis being stronger for TLM_{SPX} than TGM_{SPX}.

According to the arguments stated above, we summarize our first hypothesis relating to the S&P 500 index as:

Hypothesis 1a: *Information hypothesis in the S&P 500 index – the tail loss* (*gain*) *measure implied by S&P 500 puts* (*calls*) *will have negative* (*positive*) *predictive power on S&P 500 returns.*

Hypothesis 1b: *Risk premium hypothesis in the S&P 500 index – the tail loss* (*gain*) *measure implied by S&P 500 puts* (*calls*) *will have positive predictive power on S&P 500 returns, with this predictive relationship being stronger* (*weaker*) for the *loss* (*gain*) *measure.*

We also construct the VIX tail loss measure (TLM_{VIX}) and tail gain measure (TGM_{VIX}) for VIX OTM options. As mentioned above in the definition of VIX tail measures, the VIX level should be more economically meaningful than relative changes in the VIX. If the information hypothesis holds for the VIX, then we would expect to find TLM_{VIX} (TGM_{VIX}) being negatively (positively) correlated with the future VIX level. In the same vein, if the risk-premium hypothesis holds for the VIX, then we would expect to find both TLM_{VIX} and TGM_{VIX} being positively correlated with the future VIX level.[13] However, in contrast to the case for the S&P 500 index,

[13] Similar to the situation for the S&P 500 index, the VIX ETFs have made the VIX equivalent to a tradable asset.

given that investors are going to be more concerned by higher than lower VIX level, we would expect to find the risk-premium hypothesis being stronger for TGM_{VIX} than TLM_{VIX}. We therefore summarize our second hypothesis relating to the VIX as:

Hypothesis 2a: *Information hypothesis in the VIX – the tail loss* (*gain*) *measure implied by VIX puts* (*calls*) *will have negative* (*positive*) *predictive power on the VIX level.*

Hypothesis 2b: *Risk premium hypothesis in the VIX – the tail loss* (*gain*) *measure implied by VIX puts* (*calls*) *will have positive predictive power on the VIX level, with this predictive relationship being weaker* (*stronger*) *for the tail loss* (*gain*) *measure.*

Due to the observed negative relationship between the VIX and the S&P 500 index, it may prove worthwhile to investigate whether the option-implied tail measures for the VIX (TLM_{VIX} and TGM_{VIX}) contain information on future returns in the S&P 500 index. Chung, Tsai, Wang and Weng (2011) provided evidence to show that investors will invariably trade their price or volatility information on the S&P 500 index through both the S&P 500 and VIX options markets, whilst Park (2015) suggested that S&P 500 puts and VIX calls were useful tools for hedging the tail risk.

Based upon the information hypothesis on the VIX, as well as the negative relationship between VIX and the S&P 500 index, we hypothesize that a higher TGM_{VIX} (TLM_{VIX}) will correspond with a higher (lower) VIX level, and this will also be associated with a lower (higher) S&P 500 index level. Since this argument is based upon the information hypothesis in the VIX on the returns of the S&P 500 index, we also classify this as an 'information hypothesis'.

By contrast, if the risk-premium hypothesis holds for the VIX, given the negative relationship between the VIX and the S&P 500 index, we would expect to find a higher TLM_{VIX} or TGM_{VIX} leading to a higher VIX level, which in turn, is associated with lower S&P 500 index returns. We therefore hypothesize that with an increase TLM_{VIX} and/or TGM_{VIX}, there will be a corresponding reduction in S&P 500 index returns, with this relationship being stronger for the gain measure. Since this alternative argument is based upon the risk-premium hypothesis for the VIX, we also classify it as a 'risk-premium hypothesis'; hence, we summarize our third hypothesis as follows:

Hypothesis 3a: *Information hypothesis in the VIX for the S&P 500 Index – the tail gain* (*loss*) *measure implied by VIX calls will have negative* (*positive*) *predictive power on S&P 500 index returns.*

Hypothesis 3b: *Risk premium hypothesis of the VIX for the S&P 500 Index – the tail gain* (*loss*) *measure implied by VIX calls* (*puts*) *will have negative predictive power on S&P 500 index returns with this predictive relationship being stronger for the gain measure.*

As suggested by Figlewski and Webb (1993), option markets provide an effective venue for informed traders to realize their negative information on the underlying asset, particularly when a period of financial distress leads to more short-sales constraints in the spot markets. Therefore, given that trading activity in the options market should be more informative on market declines during periods of economic recession, we would expect to find that if the predictive relationships suggested by our hypotheses do indeed exist, they will be more robust during such periods. This leads to our fourth and final hypothesis:

Hypothesis 4: *If the predictive relationships suggested in our above hypotheses do indeed exist, then they will be found to be more robust during periods of economic recession*.

4. DATA

The daily prices of the S&P 500 index options and the VIX options used to compile our four tail measures (TLM_{SPX}, TGM_{SPX}, TLM_{VIX} and TGM_{VIX}) were obtained from the Data Express of the CBOE. As a result of differences in the availability of data, the sample period for the S&P 500 index options runs from 1 May 1997 to 31 July 2014, whilst the sample period for VIX options runs from 1 May 2006 to 31 July 2014. In other words, our sample is restricted to the shorter period when examining the joint information content of the tail measures from both options. In addition to excluding all contracts with zero bid prices, as a result of liquidity concerns, we delete any observations with a maturity period of less than 7 or more than 180 calendar days.

We follow the empirical methodology of Vilkov and Xiao (2013) on the predictive ability of S&P 500 index returns to control for certain other option-implied information, including the 'variance risk premium' (*VRP*), which is defined as the difference between (a) one-day risk-neutral variance, which is measured by the de-annualized VIX squared, $\left(\frac{VIX}{100}\right)^2 * \frac{1}{252}$ and (b) one-day 'realized variance' (RV), which is compiled from high-frequency five-minute transaction prices;[14] and model free option-implied skewness (*MFIS*) and kurtosis (*MFIK*), which are calculated based upon the method proposed by Bakshi, Kapadia and Madan (2003).

The high-frequency data on the level of the S&P 500 index were obtained from OlsenData; we use the data to compute the RV of the S&P 500 index returns, with RV also serving as a control variable in our investigation of the predictive ability of changes in the VIX. The three-month T-bill rate, which is obtained from the Federal Reserve Bank of St. Louis, is used as a proxy for the risk-free rate,[15] whilst the economic condition (recession and expansion) variables, which are used to explore the impacts of economic conditions on the predictive ability of S&P 500 index returns and the VIX level were obtained from the National Bureau of Economic

[14] We follow Bollerslev, Tauchen and Zhou (2009) to use intra-day data for the construction of the realized monthly variance. As discussed in the prior studies, including Andersen, Bollerslev, Diebold and Labys (2001) and Hansen and Lunde (2006), the selection of the sampling frequency is a trade-off between data continuity and market microstructure noises; the most frequently adopted frequency for the calculation of stock realized volatility is five minutes.

[15] The website is available at: https://research.stlouisfed.org/.

Research (NBER).[16] The summary statistics on all of the variables used in this study are reported in Panels A and B of Table 1.

<Table 1 is inserted about here>

The one-day-ahead excess return of the S&P 500 index from time *t* is defined as:

$$er_{SPX,t,t+1} = \frac{S_{t+1} - S_t}{S_t} - r_{f,t}$$

where S_t is the S&P 500 index level and $r_{f,t}$ is the three-month T-bill rate at time *t*.

Due to the longer sample period for S&P 500 index options, the variables reported in Panel A of Table 1 for our full-sample period (1 May 1997 to 31 July 2014) include the VIX and the tail loss and gain measures of the S&P 500 index, whilst the variables reported in Panel B for our sub-sample period (from 1 May 2006 to 31 July 2014) also include the tail loss and gain measures of the VIX.

The correlation coefficients between the variables, which provide preliminary observations on the predictive power of the tail gain and loss measures are reported in Tables 2a and 2b.

<Tables 2a and 2b are inserted about here>

As shown in the first column in Tables 2a and 2b, the S&P 500 one-day-ahead returns are positively related to the tail loss and gain measures implied by both the S&P 500 and VIX options, thereby indicating that either of these options-implied

[16] The data are available from the NBER website: http://www.nber.org/cycles/cyclesmain.html. Specifically, the definition of the economic recession or expansion can be found at: http://www.nber.org/cycles/ cyclesmain.html.

tail loss and gain measures is capable of providing information on the future returns of the S&P 500 index. Furthermore, when comparing the tail gain and loss measures, we find that the tail loss (gain) measure in the S&P 500 (VIX) index has a higher correlation with the S&P 500 one-day-ahead returns than the corresponding tail gain (loss) measure, which implies the likely existence of an asymmetric relationship between the left and right tail risks.

An examination of the correlations between the tail measures and the option-implied control variables reveals that the absolute correlation level between TLM_{SPX} and *VRP* is less than 0.1, which suggests that the information content of TLM_{SPX} on future S&P 500 returns may differ from that of the variance risk premium. Given that the VIX is highly correlated with almost all of the tail measures, we do not include the VIX in our list of control variables; however, since the VIX is widely regarded as an effective investor fear gauge, we also investigate whether the volatility risk and tail risk play different roles in the determination of future S&P 500 returns.

5. EMPIRICAL RESULTS

Our empirical analysis begins with an investigation into the information content of the S&P 500 option-implied tail measures on the S&P 500 future returns, followed by a comparison of the roles played by the volatility risk and tail risk, given that the VIX is found to be highly correlated with TLM_{SPX} and TGM_{SPX}. We then follow the same procedure to examine the information content of the VIX option-implied tail measures on future changes in the VIX.

Motivated by the empirically-observed correlation between the S&P 500 index and the VIX, we subsequently go on to further explore whether the tail measures implied by both the S&P 500 and VIX options provide useful information on the future returns of the S&P 500 index and whether the information contents of these two sources overlap, or whether they are totally different if both sources are actually found to provide useful information. We first of all carry out these analyses unconditionally, followed by consideration of different economic (recession and expansion) conditions.

5.1 Information Content of the Tail Measures on Future S&P 500 Returns

We investigate whether the daily excess returns in the S&P 500 index can be predicted by the tail loss and/or gain measures compiled from S&P 500 options based upon the following regression model:

$$\begin{aligned} er_{SPX,t,t+1} &= \beta_0 + \beta_1 TLM_{SPX,t} + \beta_2 TGM_{SPX,t} + \beta_3 D_t + \beta_4 TLM_{SPX,t} * D_t \\ &+ \beta_5 TGM_{SPX,t} * D_t + \beta_6 MFIS_t + \beta_7 MFIK_t + \beta_8 VRP_t + \epsilon_t, \end{aligned} \tag{12}$$

where $er_{SPX,t,t+1}$ is the one-day-ahead excess return of the S&P 500 index at time t; TLM_{SPX} (TGM_{SPX}) is the tail loss (gain) measure implied from S&P 500 deep-OTM puts (calls); and D_t is a recession dummy which takes the value of 1 for periods of economic recession, as defined by the National Bureau of Economic Research (NBER).[17] The model-free implied skewness (*MFIS*) and kurtosis (*MFIK*) and the variance risk premium (*VRP*) serve as our control variables, with the sample period running from 1 May 1997 to 31 July 2014.

The results with alternative sets of regressors are reported in Table 3, which shows that when considering only the predictive power of TLM_{SPX} in Model (1), the β_1 coefficient is found to be positively significant at the 1 per cent significance level; however, when considering economic conditions in Model (2), the β_1 coefficient turns insignificant, although the coefficient on the interaction term between TLM_{SPX} and the recession dummy, β_4, is positively significant at the 1 per cent level. The results from Models (1) and (2) imply that a higher tail loss measure leads to higher S&P 500 returns, with the effect being particularly significant during periods of economic recession.

<Table 3 is inserted about here>

Following on from our examination of the predictive power of the tail loss measure, we now turn to an investigation, in Models (3) and (4), of the predictive power of the tail gain measure, TGM_{SPX}. Similar to the finding in Model (1), when considering the predictive power of TGM_{SPX} alone, the β_2 coefficient is found to be positively significant at the 5 per cent significance level; however, no significant

[17] The two sub-periods, from 1 March 2001 to 30 November 2001 and from 1 December 2007 to 30 June 2009, are clarified as the periods of economic recession.

predictive power is discernible when economic conditions are taken into consideration, since the β_5 coefficient in Model (4) is found to be insignificant. The results from Models (3) and (4) indicate that TGM_{SPX} may also be informative on the future returns of the S&P 500 index, although no significant differences are discernible under different economic (recessionary or expansionary) conditions.

The adjusted R^2 values in Models (3) and (4) are found to be much lower than those in Models (1) and (2), thereby suggesting that the information content of TLM_{SPX} is greater than that of TGM_{SPX}. In order to further compare the predictive power of the tail loss and gain measures, we include TLM_{SPX} and TGM_{SPX} in a regression model comprising of: (i) Model (5) with no consideration of the effects of economic conditions; (ii) Model (6) with consideration of the effects of economic conditions; and (iii) Model (7) with the inclusion of the other control variables. The regression results of these models provide general confirmation of the findings in Models (1) to (4), that the predictive power of TLM_{SPX} dominates that of TGM_{SPX}.[18]

Overall, the empirical results shown in Table 3 provide general support for the risk-premium hypothesis (Hypothesis 1b) as opposed to the information hypothesis (Hypothesis 1a). In other words, in line with the findings of Kelly and Jiang (2014) and Bollerslev et al. (2015), investors holding portfolios with higher tail risk require

[18] The inconsistency between our negative coefficient on *VRP* and the finding of Bollerslev et al. (2009) is attributable to the use of different data frequencies. If we use monthly returns, then, as in Bollerslev et al. (2009), the coefficient on *VRP* turns positively significant.

compensation (that is, a positive tail risk premium). Furthermore, investors are found to be more likely to be sensitive to the tail risk during periods of economic recession, as compared to periods of economic expansion, thereby providing support for our Hypothesis 4.[19] An asymmetric risk-return relationship is also discernible since the left-tail risk is found to be more informative than the right-tail risk, which may indicate that investors are more likely to take advantage of their private information by trading in put options as opposed to call options, particularly during periods of economic recession.

5.2 Information Content of Implied Volatility vs. the Tail Measures

Both extreme positive and negative price moves will lead to an increase in volatility levels. As shown in Table 2a, the correlation coefficient between the *VIX* and TLM_{SPX} (*VIX* and TGM_{SPX}) is 0.35 (0.51), which provides us with good, sound reasoning for not including *VIX* as a control variable in our investigation of the information content of TLM_{SPX} and/or TGM_{SPX}. However, since it is already well documented that the VIX exhibits certain significant predictors of aggregate market returns or economic activities,[20] we regard it as both interesting and necessary to compare the information content of the VIX to that of the tail measures.[21]

[19] The empirical results are also in line with the theoretical model of Gourio (2012) which explains that disaster risk is related to business cycles.

[20] See Giot (2005), Banerjee, Doran and Peterson (2007) and Bekaert and Hoerova (2014).

[21] The CBOE VIX can be replaced by model-free implied volatility with the computation being

Hence, our primary goal in this sub-section is to carry out such comparisons in order to identify whether the tail risk measures contain any incremental information on the determination of future market returns. As regards the tail risk measures, we focus only on TLM_{SPX} in the following analysis, since this measure has been shown in our earlier analyses to be more informative than TGM_{SPX}. In an attempt to alleviate the potential problem of collinearity between the TLM_{SPX} and VIX, we use the orthogonal approach to obtain the residual from the TLM_{SPX} regressed on the VIX (denoted as TLM_{SPX}^{res}) in order to form the tail risk variable, since the VIX is already widely regarded as an effective investor fear gauge (Whaley, 2000; Bekaert and Hoerova, 2014). We therefore run the following regression:

$$er_{SPX,t,t+1} = \alpha + \beta_1 TLM_{SPX,t}^{res} + \beta_2 VIX_t + \beta_3 D_t + \beta_4 TLM_{SPX,t}^{res} * D_t$$
$$+\beta_5 VIX_t * D_t + \beta_6 MFIS_t + \beta_7 MFIK_t + \beta_8 VRP_t + \epsilon_t, \quad (13)$$

where all of the variables, with the exception of TLM_{SPX}^{res} (described above), are as defined in Equation (12).

As shown in Table 4, β_1 in Model (1) is found to be positively significant at the 1 per cent level, which implies that TLM_{SPX} still has predictive ability on future S&P 500 daily returns, even when the VIX effect is removed. β_4 is found to be significant in Model (2), whereas β_1 is insignificant, which is consistent with the results reported

based upon the approach of Bakshi, Kapadia and Madan (2003). The two variables are found to be almost perfectly correlated throughout our full sample period (1 May 1997 to 31 July 2014).

in Table 3, that TLM_{SPX} has predictive ability, particularly during periods of economic recession. By contrast, according to the *VIX* results in Models (3) and (4), *VIX* is generally informative for S&P 500 future returns, but this is not dependent upon economic conditions, since β_5 is found to be insignificant in Model (4).

<Table 4 is inserted about here>

In Model (5), where the *VIX* and TLM_{SPX}^{res} are simultaneously considered, the coefficients on *VIX* and TLM_{SPX}^{res} are both found to be positively significant, which means that both volatility and the tail loss risk measure are informative in the prediction of S&P 500 index returns and that their information content does not completely overlap. Furthermore, when taking into account the other control variables in Model (6), we find that β_2 and β_4 are both positively significant, which again verifies that the *VIX* is generally a useful predictor, with the predictive power of TLM_{SPX} being particularly strong during periods of economic recession. Overall, TLM_{SPX} is found to provide useful information on the determination of the future returns of the S&P 500 index, even when considering the role of the *VIX*.

5.3 Information Content of the Tail Measures on Future VIX Levels

According to our findings in the previous sub-sections, the tail loss measure implied in the S&P 500 index options is informative on the future returns of the underlying asset. In this sub-section, we go on to examine whether the predictive power of the tail risk measures also exists for other derivatives. In particular, we focus on VIX options, since trading in these volatility derivatives has the best liquidity in the world. In specific terms, we explore the information content of the tail loss measure (TLM_{VIX}) and the tail gain measure (TGM_{VIX}) implied in VIX options with regard to the one-day-ahead level of the VIX, by running the following regression:

$$\begin{aligned} VIX_{t+1} &= \beta_0 + \beta_1 TLM_{VIX,t} + \beta_2 TGM_{VIX,t} + \beta_3 D_t \\ &+ \beta_4 TLM_{VIX,t} * D_t + \beta_5 TGM_{VIX,t} * D_t + \beta_6 RV_t + \epsilon_t, \end{aligned} \quad (14)$$

where VIX_{t+1} is the CBOE VIX at time $t+1$; TLM_{VIX} (TGM_{VIX}) refers to the tail loss (gain) measure implied by VIX deep-OTM puts (calls); D_t is a recession dummy where 'recession' is as defined by the NBER; and RV_t denotes the realized variance computed from the observations from time $t-20$ to t. The sample period runs from 1 May 2006 to 31 July 2014.

In Table 5, β_1 and β_2 are found to be positively significant in all of the models with alternative sets of variables, which means that both TLM_{VIX} and TGM_{VIX} are informative in the one-day-ahead VIX index. Furthermore, when taking into account the effect of economic conditions, we find that both β_4 and β_5 are positively significant, thereby indicating that the predictive power of the tail measures is particularly strong during periods of recession, as compared to periods of expansion.

<Table 5 is inserted about here>

When comparing the predictive power of TLM_{VIX} and TGM_{VIX}, as shown in Models (6) and (7), the significance level of β_2 is found to be higher than that of β_1, whilst β_5 is found to be highly significant in both models, unlike β_4. Furthermore, the adjusted R^2 values in the models with TGM_{VIX} are found to be almost ten times those with TLM_{VIX}; for example, the value in Model (3) is 39.12 per cent, as compared to just 4.37 per cent in Model (1). These findings clearly reveal that TGM_{VIX} is more informative than TLM_{VIX} with regard to future VIX levels.

In summary, both the tail loss and gain measures implied in the VIX options are found to have positive predictive ability with regard to the level of the VIX, with this predictive power being particularly strong during periods of economic recession; this finding is clearly more consistent with the risk-premium hypothesis (Hypothesis 2b) than the information hypothesis (Hypothesis 2a), and also consistent with our Hypothesis 4. Given that we find that the information content of the tail gain measure is much greater than that of the tail loss measure, we use only the tail gain measure of the VIX in the following analysis.

5.4 Information Content of Cross-Market Tail Measures on Future S&P 500 Returns

Given that the VIX represents the 30-day implied volatility of the S&P 500 returns, and since there is an empirically-observed negative relationship between the VIX and the S&P 500 index, it is likely that the tail risk measures implied in the VIX options are also informative with regard to the future returns of the S&P 500 index. We therefore go on to further investigate whether the tail measures implied in the VIX options provide any incremental information relating to the future returns of the S&P 500 index, given the predictive power of the tail measures implied in the S&P 500 options. Since the TLM_{SPX} and TGM_{VIX} measures are found to be more informative than their corresponding opposite-direction measures on the respective future changes in the S&P 500 index and the VIX, we regress the S&P 500 one-day-ahead returns on TLM_{SPX} and TGM_{VIX} with the inclusion of the other variables of interest and the control variables, as follows:

$$\begin{aligned} er_{SPX,t,t+1} = \beta_0 + \beta_1 TLM_{SPX,t} + \beta_2 TGM_{VIX,t} + \beta_3 D_t + \beta_4 TLM_{SPX,t} * D_t \\ + \beta_5 TGM_{VIX,t} * D_t + \beta_6 MFIS_t + \beta_7 MFIK_t + \beta_8 VRP_t + \epsilon_t, \end{aligned} \tag{15}$$

where the sample period runs from 1 May 2006 to 31 July 2014 and all of the variables are as defined earlier.

As shown in Model (1) of Table 6, with no consideration of the effects of economic conditions, both β_1 and β_2 are found to be positive, although only β_1 has statistical significance, which indicates that the VIX tail gain measure has no significant predictive power on future returns in the S&P 500 index. However, when taking into account the impacts of economic conditions in Model (2), we find that β_4 is still positively significant, but β_5 is negatively significant, whilst β_1 becomes insignificant. This finding implies that both tail measures are informative with regard to the future returns of the S&P 500 index during periods of economic recession, with opposite predictive directions, which is consistent with the joint effect predicted by both Hypothesis 3a (the information hypothesis) and Hypothesis 4.

<Table 6 is inserted about here>

6. FURTHER DISCUSSION

6.1 Predictive Ability across Alternative Time Horizons

As our empirical results clearly demonstrate the predictive ability of the TLM_{SPX} and TGM_{VIX} on one-day-ahead returns in the S&P 500 index, we go on to carry out further analysis aimed at determining whether such predictive power is found to persist over longer periods. In specific terms, we modify Equation (15) by replacing the dependent variable with the weekly or monthly returns of the S&P 500 index.

The results are reported in Table 7, where our specific focus is on the coefficients of the two tail risk measures. As the table shows, after controlling for the effects of *MFIS*, *MFIK* and *VRP*, β_1 and β_2 are found to be statistically insignificant for both the weekly and monthly returns; this finding would seem to clearly indicate that the predictive power of the tail loss and gain measures on market returns is only short-lived.

<Table 7 is inserted about here>

6.2 Predictive Ability on Extreme Changes

Since the tail measures are theoretically related to occurrences of extreme changes in the asset prices, we are also interested in whether the tail loss and gain measures contain information relating to occurrences of extreme events in the market. We modify Equation (15) by replacing the dependent variable with a dummy variable, $EX_{SPX,t,t+1}$, to indicate whether the one-day-ahead market return at time t ($er_{SPX,t,t+1}$, which is defined above), is an extreme event, according to the following definition.

$$EX_{SPX,t,t+1} = \begin{cases} 1, & er_{SPX,t,t+1} < -3\sigma_t \\ 0, & otherwise, \end{cases}$$

where σ_t is defined as the realized variance measured by five-minute frequency returns of the S&P 500 index at time t. The regression results, with the inclusion of the alternative sets of explanatory variables, are reported in Table 8.

<Table 8 is inserted about here>

Individually, β_1 and β_2 in Models (1) and (3) are found to be negatively significant, which indicates that both the TLM_{SPX} and TGM_{VIX} measures reduce the likelihood of extremely negative one-day-ahead returns. Similarly, β_4 in Model (2) and β_5 in Model (4) are both found to be negative, which indicates that the reduced likelihood predicted by the two tail measures is stronger during periods of economic recession, although none of the coefficients are significant. When simultaneously considering both tail measures in Models (5) and (6), we find that the TLM_{SPX} measure performs better than the TGM_{VIX} measure, since only β_1 is found to be statistically significant.

Overall, our findings suggest that the higher the TLM_{SPX} and/or TGM_{VIX}, the lower the likelihood of extremely negative S&P 500 returns, which is consistent with our findings on the risk-premium hypothesis in the main analysis, that the higher the TLM_{SPX} measure, the higher the future S&P 500 returns.

7. CONCLUSIONS

We use a deep-OTM option-pricing formula in this study (originally proposed by Hamidieh, 2011) and follow the methodology of Vilkov and Xiao (2013) to examine whether the option-implied tail-risk measures provide any information content with predictive ability on the future dynamics of the underlying asset. Our empirical results, based upon the tail risk measures compiled from the S&P 500 index and the VIX options, indicate that both tail-risk measures implied by the two option indices have positive predictive ability on future returns, as well as the level of their underlying assets. This suggests that investors are generally risk-averse to the tail risk, and thus, require compensatory premiums, which is consistent with the findings of the prior studies (Kelly and Jiang, 2014; Bollerslev et al., 2015).

Furthermore, the tail-risk measures implied by the two option indices jointly predict the returns of the S&P 500 index, with our findings indicating that the S&P

500 implied measures are more powerful, whilst the price impact of the option-implied tail risks are found to be stronger during periods of economic recession, as compared to periods of economic expansion. This implies that the business cycle affects the predictive ability of the tail risk on stock market returns, which is consistent with the theoretical results of Gourio (2012). Finally, the impact of these tail measures is found to be only short-lived.

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Table 1 Summary statistics

This table reports the summary statistics and correlation coefficients of S&P 500 daily excess returns, CBOE VIX, and tail loss and gain measures implied by S&P 500 and VIX options (TLM_{SPX}, TGM_{SPX}, TLM_{VIX}, and TGM_{VIX}). The full period runs from 1 May 1997 to 31 July 2014, while the sub-period runs from 1 May 2006 to 31 July 2014.

Variables	No. of Obs.	Mean	S.D.	Skewness	Kurtosis	AR(1)
Panel A: Full Period						
$er_{SPX,t,t+1}$	4,341	0.0002	0.0128	–0.0340	10.5557	–0.0764
VIX	4,341	0.2166	0.0867	1.8585	9.1051	0.9808
TLM_{SPX}	4,341	0.0638	0.0336	1.5817	12.1168	0.3353
TGM_{SPX}	4,328	0.0211	0.0134	1.6535	7.9604	0.4531
Panel B: Sub-period						
$er_{SPX,t,t+1}$	2,078	0.0002	0.0138	–0.0733	12.9261	–0.0620
VIX	2,078	0.2148	0.1038	2.1264	8.7770	0.9816
TLM_{SPX}	2,078	0.0685	0.0371	1.8199	14.0835	0.3751
TGM_{SPX}	2,078	0.0184	0.0130	2.1320	10.9128	0.4705
TLM_{VIX}	2,078	11.3094	4.3018	–0.2320	2.3209	0.9395
TGM_{VIX}	2,078	6.9626	3.2306	1.1189	5.2652	0.6747

Table 2a Full period correlation matrix

Tables 2a and 2b report the correlation coefficients on S&P 500 daily excess returns, CBOE VIX and tail loss and gain measures implied by S&P 500 and VIX options (TLM_{SPX}, TGM_{SPX}, TLM_{VIX}, and TGM_{VIX}). The full period runs from 1 May 1997 to 31 July 2014, whilst the sub-period runs from 1 May 2006 to 31 July 2014.

Variables	$er_{SPX,t,t+1}$	TLM_{SPX}	TGM_{SPX}	VIX	MFIS	MFIK
TLM_{SPX}	0.0484	–				
TGM_{SPX}	0.0298	0.2119	–			
VIX	0.0419	0.3532	0.5155	–		
MFIS	0.0225	–0.2435	0.2455	0.2184	–	
MFIK	–0.0125	0.2016	–0.2260	–0.3038	–0.8968	–
VRP	–0.0396	–0.0175	0.1039	0.0912	–0.0157	–0.0186

Table 2b Sub-period correlation matrix

Variables	$er_{SPX,t,t+1}$	TLM_{SPX}	TGM_{SPX}	TLM_{VIX}	TGM_{VIX}	VIX	MFIS	MFIK
TLM_{SPX}	0.0786	–						
TGM_{SPX}	0.0410	0.3109	–					
TLM_{VIX}	0.0171	0.3122	0.2540	–				
TGM_{VIX}	0.0375	0.3728	0.4405	0.2701	–			
VIX	0.0396	0.3775	0.6177	0.2112	0.6401	–		
MFIS	0.0222	–0.1821	0.1974	–0.1616	–0.0050	0.3186	–	
MFIK	–0.0150	0.1722	–0.1555	0.1739	–0.0176	–0.3299	–0.8945	–
VRP	–0.0465	–0.0723	0.0752	0.1363	–0.0079	–0.0015	–0.0196	0.0069

Table 3 The effect of the information content of the S&P 500 tail risk measures on future S&P 500 returns

This table reports the results obtained from the following regression models and daily observations:

$$er_{SPX,t,t+1} = \alpha + \beta_1 TLM_{SPX,t} + \beta_2 TGM_{SPX,t} + \beta_3 D_t + \beta_4 TLM_{SPX,t} * D_t + \beta_5 TGM_{SPX,t} * D_t + \beta_6 MFIS_t + \beta_7 MFIK_t + \beta_8 VRP_t + \epsilon_t,$$

where $er_{SPX,t,t+1}$ is the one-day-ahead excess return of the S&P 500 index at time t; $TLM_{SPX,t}$ ($TGM_{SPX,t}$) is the tail loss (gain) measure implied by the deeper OTM puts (calls) in the S&P 500 index; and D_t is a recession dummy variable which takes the value of 1 if the observations occurred during either of the recession periods (March 2001 to November 2001 and December 2007 to June 2009), as defined by the NBER. The control variables include model-free option-implied skewness (*MFIS*) and kurtosis (*MFIK*) as defined by Bakshi et al. (2003) and the variance risk premium (*VRP*) defined as the difference between the VIX index and realized variance constructed from the five-minute frequency returns of the S&P 500 index. The sample period runs from 1 May 1997 to 31 July 2014.

Variables	Model (1)		Model (2)		Model (3)		Model (4)		Model (5)		Model (6)		Model (7)	
	Coeff	t-stat.	Coeff	t-stat.	Coeff	t-stat.	Coeff	t-stat.	Coeff	t-stat.	Coeff	t-stat.	Coeff	t-stat.
β_1	0.0178***	3.08	0.0010	0.15	–	–	–	–	0.0163***	2.75	–0.0001	–0.01	0.0063	0.93
β_2	–	–	–	–	0.0288**	1.98	0.0249	1.46	0.0201	1.36	0.0249	1.46	0.0129	0.72
β_3	–	–	–0.0088***	–7.16	–	–	–0.0028***	–2.58	–	–	–0.0080***	–5.91	–0.0083***	–6.07
β_4	–	–	0.1035***	6.84	–	–	–	–	–	–	0.1088***	6.63	0.1022***	6.13
β_5	–	–	–	–	–	–	0.0513	1.47	–	–	–0.0474	–1.27	–0.0305	–0.81
β_6	–	–	–	–	–	–	–	–	–	–	–	–	0.0017**	2.12
β_7	–	–	–	–	–	–	–	–	–	–	–	–	0.0001	1.05
β_8	– –	–	–	–	–	–	–	–	– –	–	–	–	–0.0054	–1.31
β_0	–0.0009**	–2.26	0.0003	0.64	–0.0004	–1.16	–0.0002	–0.39	–0.0013***	–2.67	–0.0001	–0.28	0.0018**	2.04
No. of Obs.	4,341		4,341		4,328		4,328		4,328		4,328		4,319	
Adj. R^2 (%)	0.20		1.34		0.07		0.22		0.22		1.36		1.52	

Table 4   Comparison of the information content in the VIX and the tail risk measures

This table reports the results obtained from the following regression models and daily observations:

$$er_{SPX,t,t+1} = \alpha + \beta_1 TLM^{res}_{SPX,t} + \beta_2 VIX_t + \beta_3 D_t + \beta_4 TLM^{res}_{SPX,t} * D_t + \beta_5 VIX_t * D_t + \beta_6 MFIS_t + \beta_7 MFIK_t + \beta_8 VRP_t + \epsilon_t,$$

where $er_{SPX,t,t+1}$ is the one-day-ahead excess return of the S&P 500 index at time t; $TLM^{res}_{SPX,t}$ is the residual from regressing $TLM_{SPX,t}$ on VIX_t (where VIX_t denotes the volatility index provided by CBOE at time t); and D_t is a recession dummy variable which takes the value of 1 if the observations occurred during either of the recession periods (March 2001 to November 2001 and December 2007 to June 2009), as defined by the NBER. The control variables include model-free option-implied skewness (*MFIS*) and kurtosis (*MFIK*) as defined by Bakshi et al. (2003) and the variance risk premium (*VRP*) defined as the difference between the VIX index and realized variance constructed from the five-minute frequency returns of the S&P 500 index. The sample period runs from 1 May 1997 to 31 July 2014.

Variables	Model (1)		Model (2)		Model (3)		Model (4)		Model (5)		Model (6)	
	Coeff	t-stat.	Coeff	t-stat.	Coeff	t-stat.	Coeff	t-stat.	Coeff	t-stat.	Coeff	t-stat.
β_1	0.0188***	2.85	0.0020	0.29	–	–	–	–	0.0168**	2.55	0.0008	0.12
β_2	–	–	–	–	0.0073***	3.07	0.0114***	3.38	0.0067***	2.79	0.0103***	3.06
β_3	–	–	–0.0010*	–1.69	–	–	–0.0030*	–1.86	–	–	–0.0007	–0.45
β_4	–	–	0.1336***	7.19	–	–	–	–	–	–	0.1268***	6.68
β_5	–	–	–	–	–	–	0.0013	0.24	–	–	–0.0042	–0.78
β_0	0.0021***	2.97	0.0024***	3.31	–0.0002	–0.24	–0.0003	–0.29	0.0004	0.48	0.0002	0.24
Controls	Yes		Yes		Yes		Yes		Yes		Yes	
No. of Obs.	4,330		4,330		4,330		4,330		4,330		4,330	
Adj. R^2 (%)	0.32		1.60		0.35		0.70		0.48		1.81	

Table 5 The effect of the information content of VIX tail risk measures on future VIX levels

This table presents the results based upon the following regression model and daily observations:

$$VIX_{t+1} = \beta_0 + \beta_1 TLM_{VIX,t} + \beta_2 TGM_{VIX,t} + \beta_3 D_t + \beta_4 TLM_{VIX,t} * D_t + \beta_5 TGM_{VIX,t} * D_t + \beta_6 RV_t + \epsilon_t$$

where VIX_{t+1} is the VIX index at time t+1; $TLM_{VIX,t}$ ($TGM_{VIX,t}$) is the tail loss (gain) measure implied by the deeper OTM put (call) options in the VIX index; D_t is a recession dummy variable which takes the value of 1 if the observations occurred during either of the recession periods (March 2001 to November 2001 and December 2007 to June 2009), as defined by the NBER; and RV_t is a control variable for the realized variance constructed from five-minute frequency returns of the S&P 500 index from time t–20 to t. The sample period runs from 1 May 2006 to 31 July 2014.

Variables	Model (1)		Model (2)		Model (3)		Model (4)		Model (5)		Model (6)		Model (7)	
	Coeff	t-stat.	Coeff	t-stat.	Coeff	t-stat.	Coeff	t-stat.	Coeff	t-stat.	Coeff	t-stat.	Coeff	t-stat.
β_1	0.0051***	9.79	0.0052***	12.06	–	–	–	–	0.0011**	2.50	0.0035***	10.98	0.0033***	13.58
β_2	–	–	–	–	0.0201***	36.54	0.0109***	22.17	0.0197***	34.55	0.0094***	18.96	0.0054***	13.68
β_3	–	–	0.0480***	3.86	–	–	–0.0003	–0.04	–	–	0.0033	0.35	–0.0082	–1.14
β_4	–	–	0.0111***	9.97	–	–	–	–	–	–	0.0000	0.01	0.0021***	3.15
β_5	–	–	–	–	–	–	0.0175***	21.68	–	–	–0.0178***	21.31	–0.0088***	12.89
β_6	–	–	–	–	–	–	–	–	–	–	–	–	0.4284***	38.04
β_0	0.1576***	25.15	0.1256***	23.79	0.0749***	17.74	0.1119***	31.26	0.0655***	11.58	0.0817***	18.48	0.0602***	17.62
No. of Obs.	2,077		2,077		2,077		2,077		2,077		2,077		2,075	
Adj. R^2 (%)	4.37		44.82		39.12		70.20		39.27		72.05		83.72	

Table 6 The effect of the information content of S&P 500 and VIX tail risk measures on future S&P 500 returns

This table reports the results obtained from the following regression models and daily observations:

$$er_{SPX,t,t+1} = \beta_0 + \beta_1 TLM_{SPX,t} + \beta_2 TGM_{VIX,t} + \beta_3 D_t + \beta_4 TLM_{SPX,t} * D_t$$
$$+\beta_5 TGM_{VIX,t} * D_t + \beta_6 MFIS_t + \beta_7 MFIK_t + \beta_8 VRP_t + \epsilon_t$$

where $er_{SPX,t,t+1}$ is the one-day-ahead excess return of the S&P 500 index at time t; $TLM_{SPX,t}$ ($TGM_{VIX,t}$) is the tail loss (gain) measure implied by the deeper OTM puts (calls) in the S&P 500 (VIX) index; and D_t is a recession dummy variable which takes the value of 1 if the observations occurred during either of the recession periods (March 2001 to November 2001 and December 2007 to June 2009), as defined by the NBER. The control variables include model-free option-implied skewness (*MFIS*) and kurtosis (*MFIK*) as defined by Bakshi et al. (2003) and the variance risk premium (*VRP*) defined as the difference between the VIX index and realized variance constructed from the five-minute frequency returns of the S&P 500 index. The sample period runs from 1 May 2006 to 31 July 2014.

Variables	Model (1)		Model (2)	
	Coeff.	t-stat.	Coeff.	t-stat.
β_1	0.0292***	3.27	0.0076	0.77
β_2	0.0036	0.35	0.0018	0.15
β_3	–	–	-0.0101***	-5.47
β_4	–	–	0.1410***	5.93
β_5	–	–	-0.0410*	-1.73
β_0	0.0000	0.01	0.0041***	2.64
Controls	Yes		Yes	
No. of Obs.	2,076		2,076	
Adj. R^2 (%)	0.69		2.99	

Table 7 Predictive ability across different time horizons

This table presents the results based upon the following regression model and daily observations:

$$er_{SPX,t,t+s} = \beta_0 + \beta_1 TLM_{SPX,t} + \beta_2 TGM_{VIX,t} + \beta_3 D_t + \beta_4 TLM_{SPX,t} * D_t + \beta_5 TGM_{VIX,t} * D_t$$
$$+\beta_6 MFIS_t + \beta_7 MFIK_t + \beta_8 VRP_t + \epsilon_t,$$

where $er_{SPX,t,t+s}$ is the s-day-ahead excess return of the S&P 500 index at time t; $TLM_{SPX,t}$ ($TGM_{VIX,t}$) is the tail loss (gain) measure implied by the deeper OTM puts (calls) in the S&P 500 (VIX) index; and D_t is a recession dummy variable which takes the value of 1 if the observations occurred during either of the recession periods (March 2001 to November 2001 and December 2007 to June 2009), as defined by the NBER. The control variables include model-free option-implied skewness (*MFIS*) and kurtosis (*MFIK*) as defined by Bakshi et al. (2003) and the variance risk premium (*VRP*) defined as the difference between the VIX index and realized variance constructed from the five-minute frequency returns of the S&P 500 index. The sample period runs from 1 May 1997 to 31 July 2014.

Variables	Weekly (s=5)				Monthly (s=20)			
	Model (1)		Model (2)		Model (3)		Model (4)	
	Coeff.	t-stat.	Coeff.	t-stat.	Coeff.	t-stat.	Coeff.	t-stat.
β_1	–0.0162	–0.53	–	–	–0.0370	–0.31	–	–
β_2	–	–	–0.0004	–0.65	–	–	0.0005	0.24
β_0	0.0020	0.56	0.0016	0.21	–0.0045	–0.32	–0.0068	–0.22
Controls	Yes		Yes		Yes		Yes	
No. of Obs.	864		415		217		104	
Adj. R^2 (%)	0.48		0.45		0.42		–1.61	

Table 8 Predictive ability on extreme changes

This table presents the results based upon the following regression model and daily observations:

$$EX_{SPX,t,t+1} = \beta_0 + \beta_1 TLM_{SPX,t} + \beta_2 TGM_{VIX,t} + \beta_3 D_t + \beta_4 TLM_{SPX,t} * D_t + \beta_5 TGM_{VIX,t} * D_t + \beta_6 MFIS_t + \beta_7 MFIK_t + \beta_8 VRP_t + \epsilon_t$$

where $EX_{SPX,t,t+1}$ is a dummy variable which takes the value of 1 if the one-day ahead S&P 500 excess daily return at time t ($er_{SPX,t,t+1}$) is lower than minus three standard deviations measured by the daily realized variance; all other variables are defined in the previous tables. The sample period runs from 1 May 1997 to 31 July 2014.

Variables	Model (1)		Model (2)		Model (3)		Model (4)		Model (5)		Model (6)	
	Coeff	t-stat.	Coeff	t-stat.	Coeff	t-stat.	Coeff	t-stat.	Coeff	t-stat.	Coeff	t-stat.
β_1	–4.1549**	–2.49	–3.4705**	–1.99	–	–	–	–	–5.8821***	–2.62	–5.0758**	–2.15
β_2	–	–	–	–	–0.0418*	–1.80	–0.0281	–1.12	–0.0284	–1.17	–0.0203	–0.78
β_3	–	–	0.6273**	–2.02	–	–	0.7652*	1.84	–	–	0.9644**	2.17
β_4	–	–	–7.5038	–1.62	–	–	–	–	–	–	–8.1712	–1.19
β_5	–	–	–	–	–	–	–0.0892	–1.52	–	–	–0.0415	–0.62
β_0	–0.5724**	–2.40	–0.6190**	–2.51	–0.5938*	–1.69	–0.7577*	–1.94	–0.6990**	–1.97	–0.9118**	–2.29
Controls	Yes		Yes		Yes		Yes		Yes		Yes	
No. of Obs.	4,330		4,330		2,076		2,076		2,076		2,076	